Emulsions in HIT, an insight on multiphase turbulence

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Introduction: some fun example

Introduction: some *less* fun example

Introduction: fundamental multiphase turbulence

- Emulsions allow to study multiphase turbulence in simplified conditions, focusing on the role of the interface

- In homogeneous isotropic turbulence (HIT), emulsions can be more easily compared against single-phase turbulence: modulation of turbulence cascade

- The configuration allows to analyze the interaction between turbulence and droplet formation

- Emulsions allow to design numerical experiments to understand the role of fluid viscosity contrast, volume fraction and surface tension.

How does the interaction between droplet breakup and turbulence occur?

Introduction: fundamental multiphase turbulence

We solve the one-fluid formulation of the Navier-Stokes equation for an incompresible flow in a box of size $L = 2\pi$:

$$\partial_i u_i = 0$$

$$\rho(\partial_t u_i + u_j \partial_j u_i) = -\partial_i p + \partial_i \left[\mu(\partial_i u_j + \partial_j u_i) \right] + \sigma \xi \delta_s n_i + f_i^T$$

The Volume of Fluid (VOF) MTHINC method is used.

Finally, turbulence is sustained by the Arnold-Beltrami-Childress (ABC) forcing:

$$f_x^T = A\, sin\, \kappa_0 z + C\, cos\, \kappa_0 y$$
$$f_y^T = B\, sin\, \kappa_0 x + A\, cos\, \kappa_0 z$$
$$f_z^T = C\, sin\, \kappa_0 y + B\, cos\, \kappa_0 x\, .$$

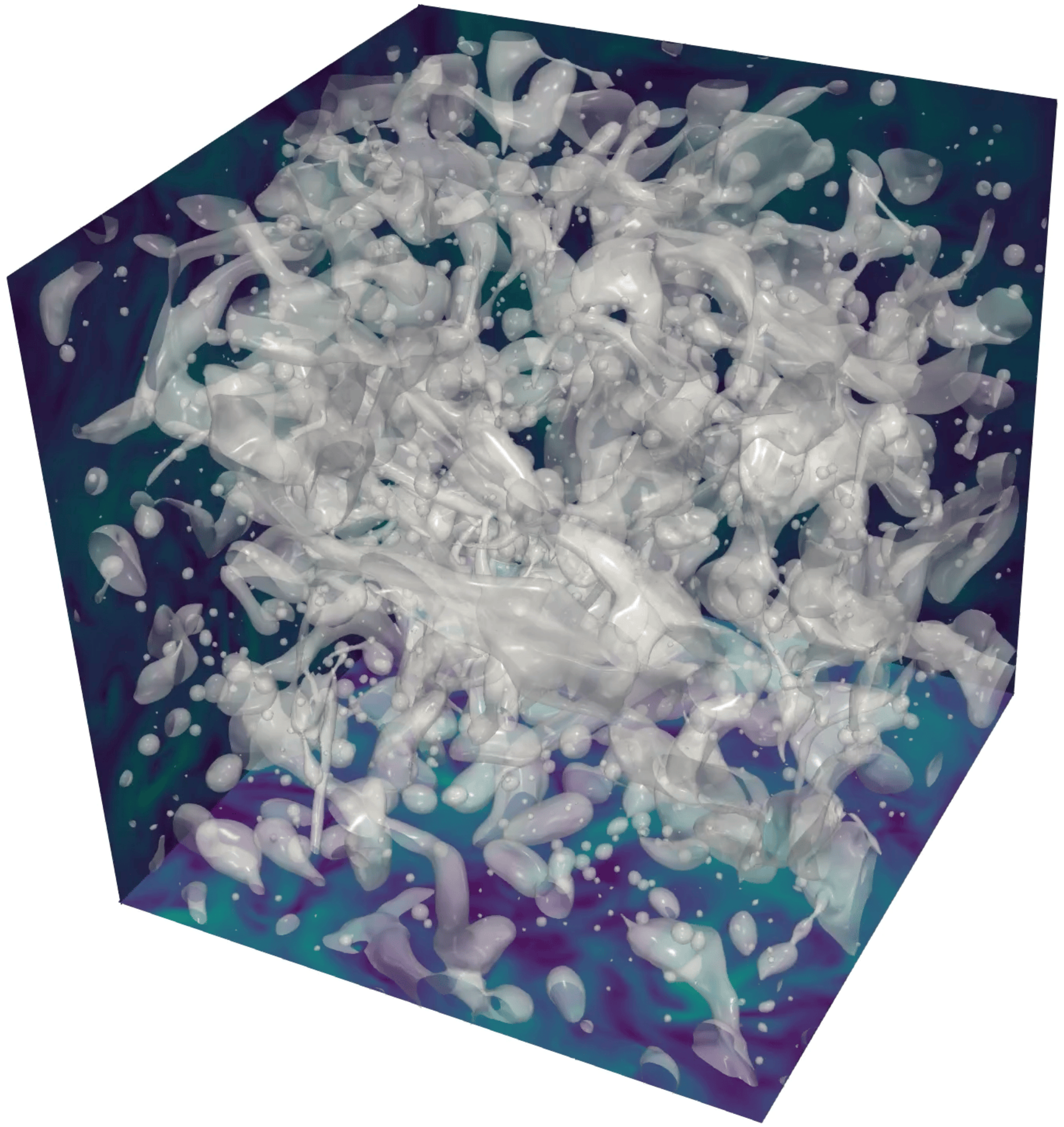
With $A = B = C = 1$

Numerical setup

- Tri-periodic box of $L = 2\pi$, with a grid of 512^3 points

- Forcing on $\kappa_0 = 2\pi/\mathscr{L} = 2$

- Droplets initialized on a single-phase turbulent field with $Re_\lambda = 137$



- Volume fraction $\alpha \in [0.03,0.5]$, with $We_{\mathscr{L}} = 42.6$ and $\mu_d/\mu_c = 1$

- Viscosity ratio $\mu_d/\mu_c \in [0.01,100]$, with $We_{\mathscr{L}} = 42.6$ and $\alpha = 0.1$

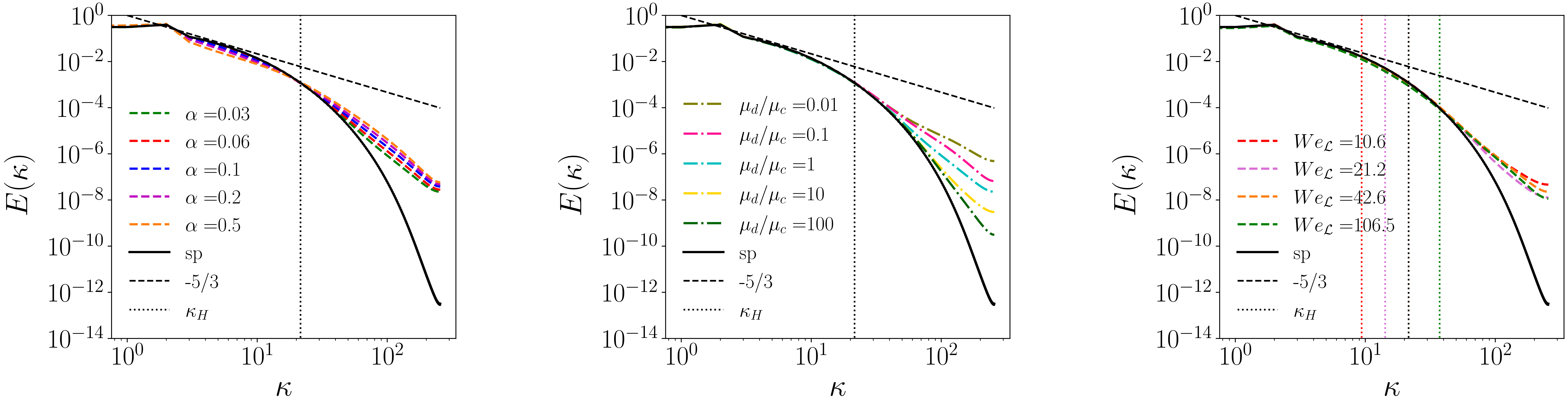
- Weber number $We_{\mathscr{L}} = \dfrac{\rho u^2_{rms,sp}\mathscr{L}}{\sigma} \in [10.6,106.5]$, with $\alpha = 0.03$ and $\mu_d/\mu_c = 1$

Turbulence modulation

Crialesi-Esposito, M., Rosti, M., Chibbaro, S., & Brandt, L. (2022). **Modulation of homogeneous and isotropic turbulence in emulsions**. *Journal of Fluid Mechanics, 940*, A19.

Crialesi-Esposito, M., Boffetta, G., Brandt, L., Chibbaro, S., & Musacchio, S. (2023). **Intermittency in turbulent emulsions**. *arXiv preprint arXiv:2301.01537*.

Comparison of one-dimensional energy spectra



- For all multiphase cases, energy decreases at large scale and increase at small scales
- For all cases $d_H = 0.725\epsilon^{-2/5}(\rho/\sigma)^{-3/5}$ is an approximate descriptor of the pivoting point
- Variation of viscosity (especially for $\mu_d/\mu_c < 1$) gives the largest variation of energy at small scales

Scale-by-Scale energy balance

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p/\rho + \partial_i \left[\nu(\partial_i u_j + \partial_j u_i)\right] + \sigma\xi\delta_s n_i/\rho + f_i^T/\rho$$

$$\partial_t \tilde{u}_i + \widetilde{G}_i = -i\kappa p/\rho + \widetilde{V}_i + \tilde{f}_i^\sigma + \tilde{f}_i^T/\rho$$

$$\partial_t E(\kappa_i) + T(\kappa_i) = +\mathscr{D}(\kappa_i) + \mathscr{S}_\sigma(\kappa_i) + \mathscr{F}(\kappa_i)$$

$E = (\tilde{u}_i \tilde{u}_i^*)$ is the kinetic energy in the spectral domain;

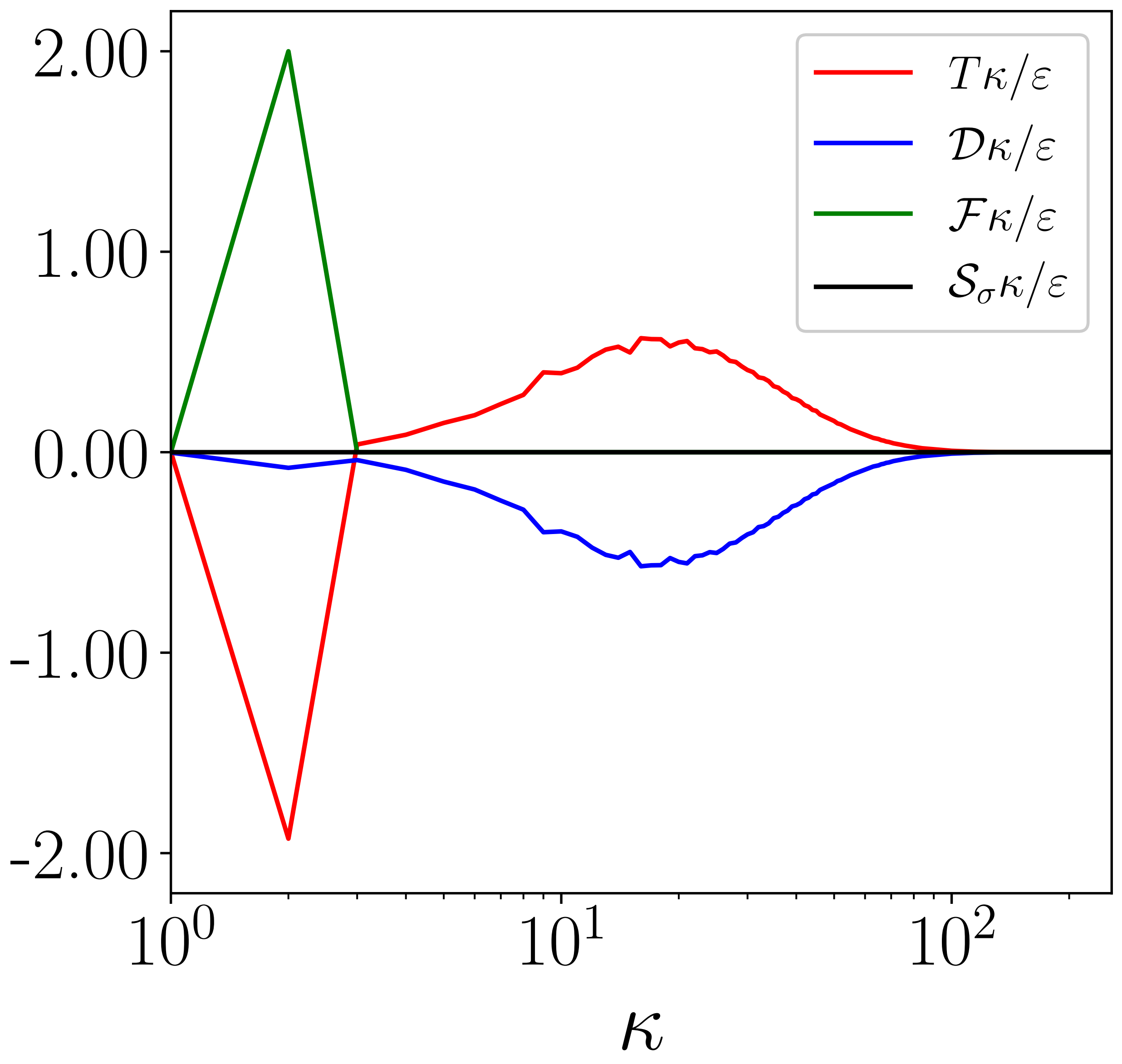
$T = (\tilde{G}_i \tilde{u}_i^* + \tilde{G}_i^* \tilde{u}_i)$ is the energy transfer due to the non-linear term;

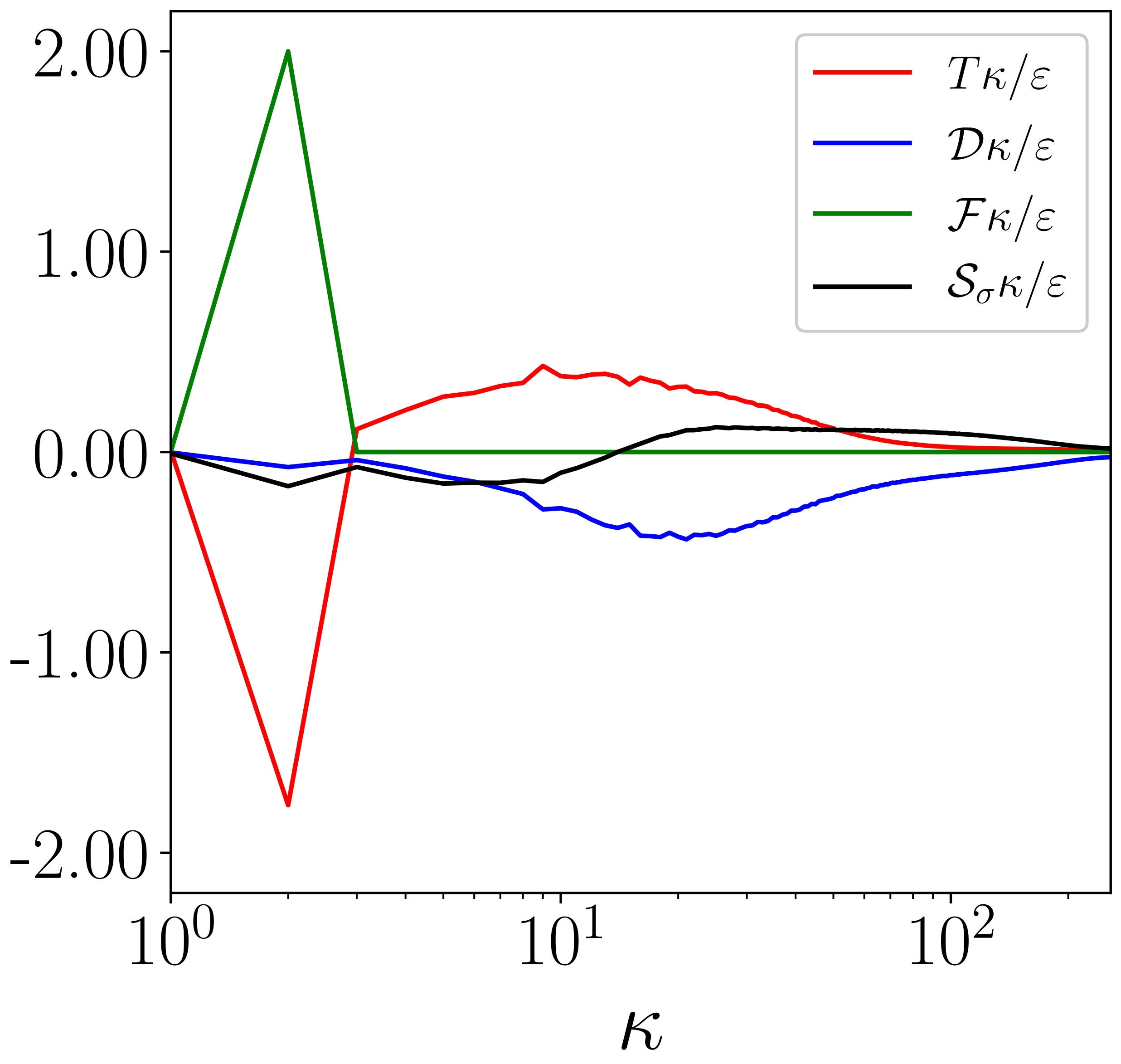
$\mathscr{D} = (\tilde{V}_i \tilde{u}_i^* + \tilde{V}_i^* \tilde{u}_i)$ is the viscous dissipation;

$\mathscr{S}_\sigma = (\tilde{f}_{\sigma_i} \tilde{u}_i^* + \tilde{f}_{\sigma_i}^* \tilde{u}_i)$ is the surface tension energy transfer at the different scales;

$\mathscr{F} = (\tilde{f}_i \tilde{u}_i^* + \tilde{f}_i^* \tilde{u}_i)$ is the energy input due to the large-scale forcing.

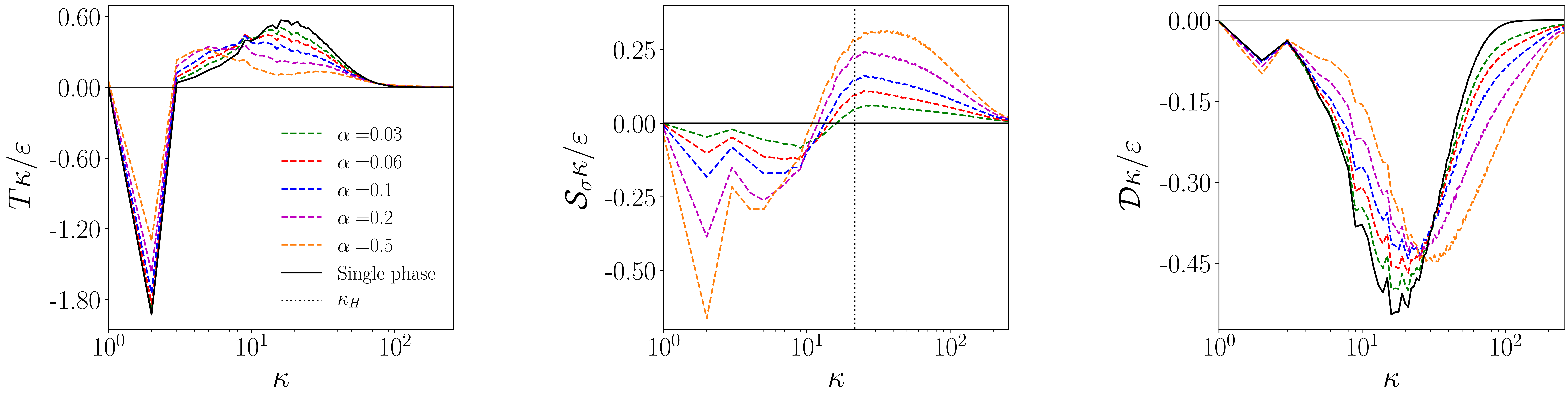
SBS budget in multiphase flows





- In single-phase, energy is transported by non-linear term and absorbed by the viscous dissipation.
- In multiphase, surface tension absorbs energy at large scales and injects it at small scales
- Energy is transported by surface tension deep into the dissipative range, forcing viscous dissipation to act at smaller scales.

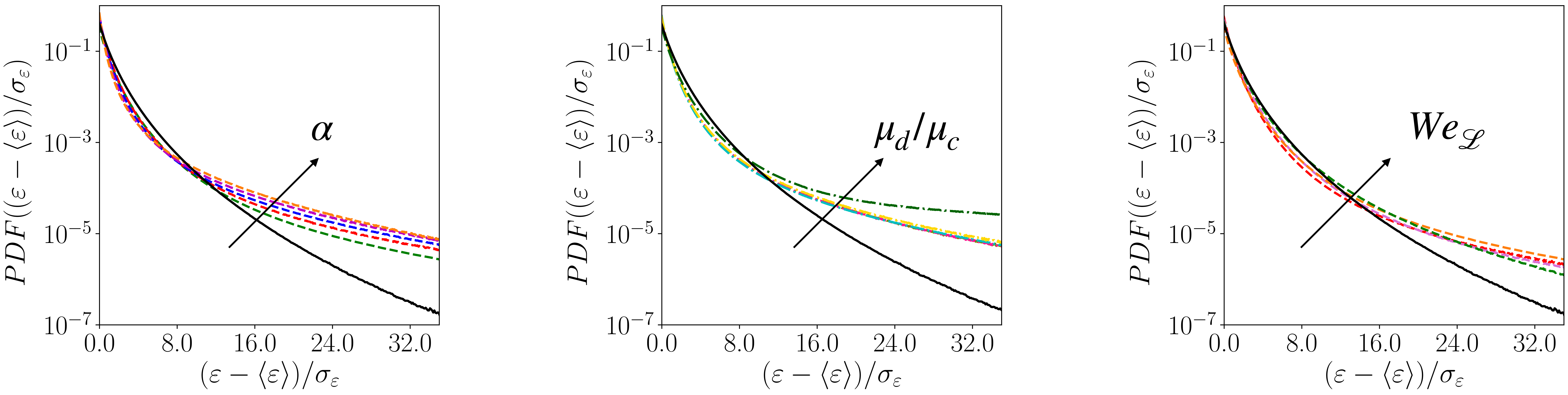
Influence of volume fraction on SBS



- As α increases, interfacial area increases as well, energy transport through surface tension increases

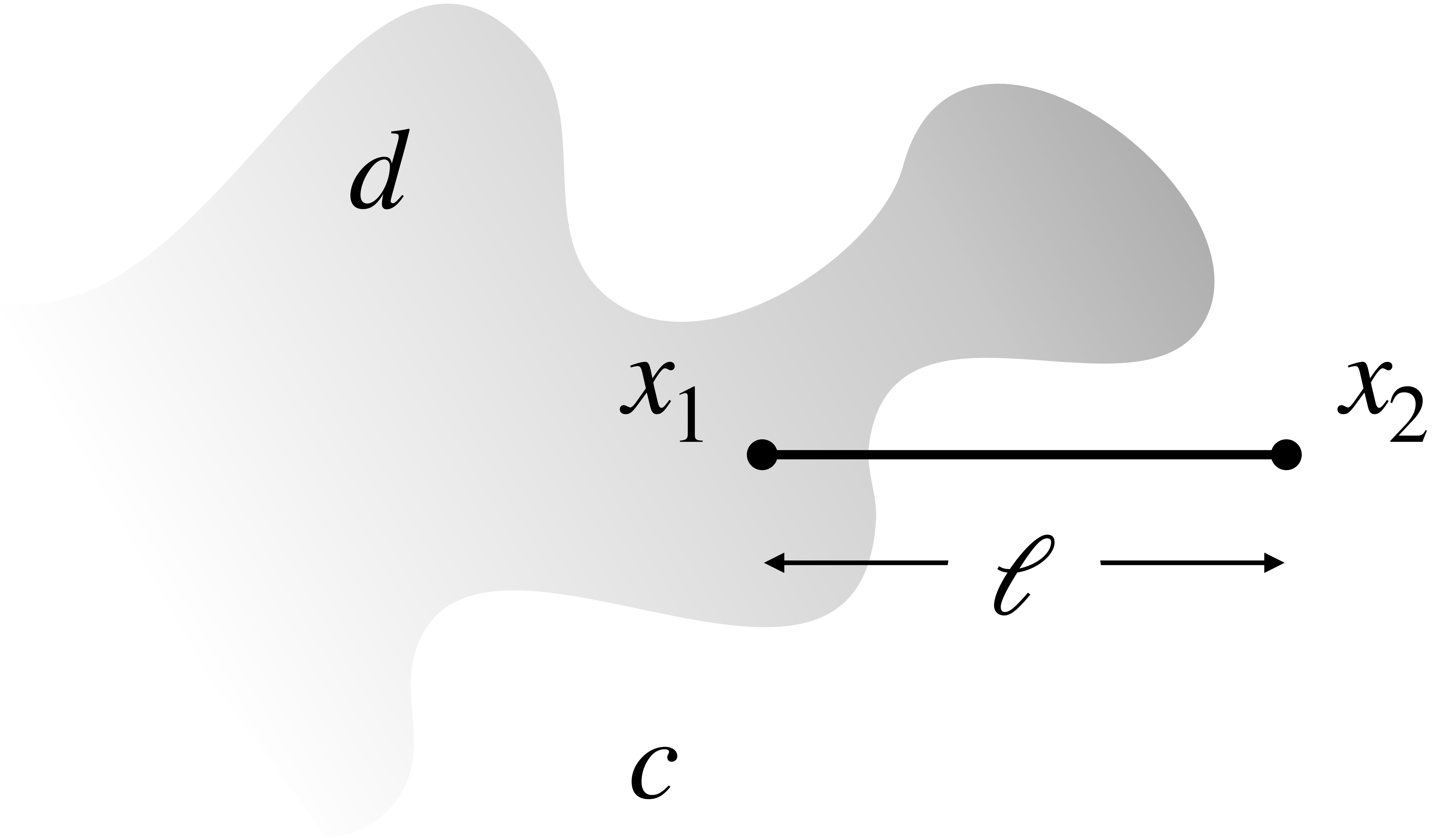
- The higher α, the more energy is transferred to small scales, extending the dissipation range

Liquid interface effects on intermittency



- All cases with emulsion show strong intermittent behaviour with respect to the single-phase
- Of all parameterisation, α shows the strongest influence
- Similar trends are also observed on vorticity

The behaviour of velocity increments across the interface



$$\delta u_\ell = u(x_2) - u(x_1) = u(x+\ell) - u(x)$$

Where x_i could be in any phase

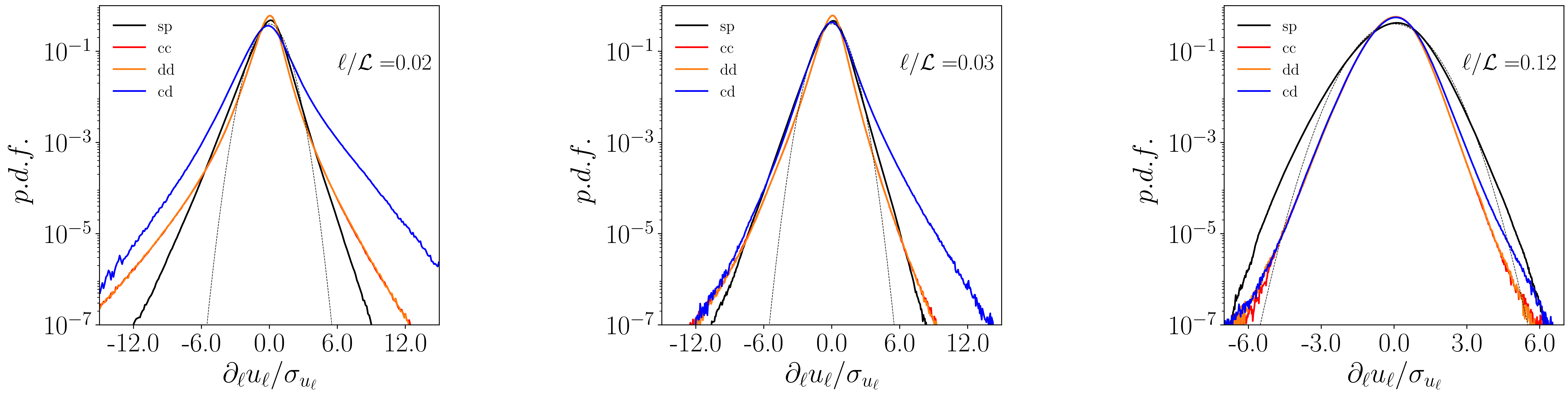
$$\delta u_\ell|_{mn} = u(x_2|_m) - u(x_1|_n)$$

Where x_1, x_2 are bounded to be in a specific phase (cc, dd) or across the interphase (cd)

We can decompose the behaviour of velocity increments on each phase and across the interface

Velocity increments at different scales ($\alpha = 0.5$)

* All data are normalised with the single-phase σ_{u_ℓ} at the corresponding scale



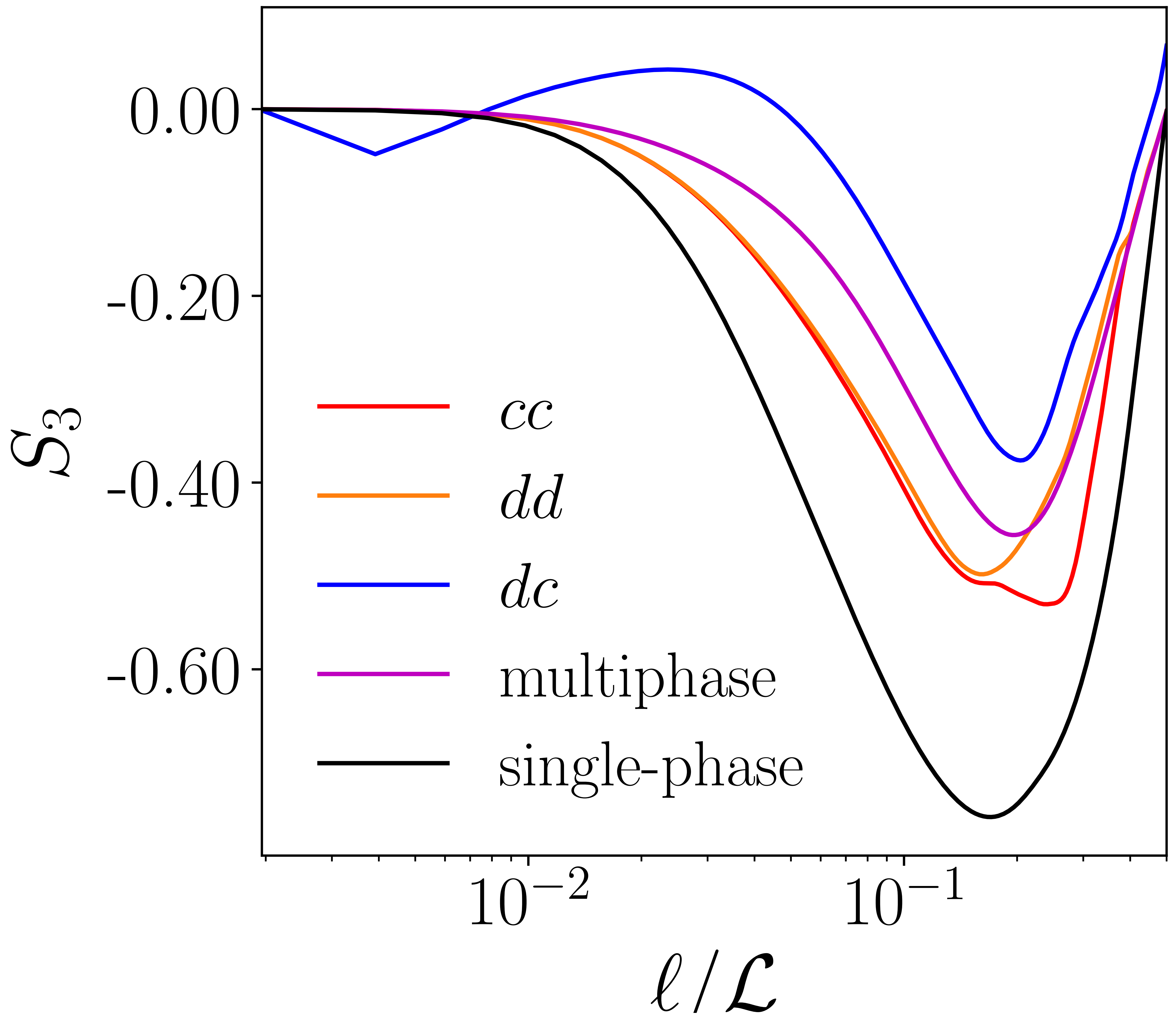
- At small scales, velocity gradients are higher in multiphase, with strongly intermittent gradients across the interface

- Towards large scales, *mp* gradients decrease (consistently with reduced energy transport from the non-linear term), while still the interface shows stronger gradients.

- While *sp*, *cc*, *dd* are always negative-skewed, the interface seems to show positive skewness

Interface effects on non-linear transport ($\alpha = 0.5$)

$$S_n = \langle (\delta u)^n \rangle$$



- Under the hypothesis of HIT, S_3 is analogous of the energy fluxes in spectral space
- The interface shows positive values, which in turns decrease the total fluxes of the multiphase flow
- Inside each phase, the behaviour of the S_3 is consistent, still quite different compared to *sp*.
- Ultimately, the total non-linear flux is decrease both due to alterations within each phase, but also due to positive strong gradients at the interface.

Conclusions on turbulence modulation

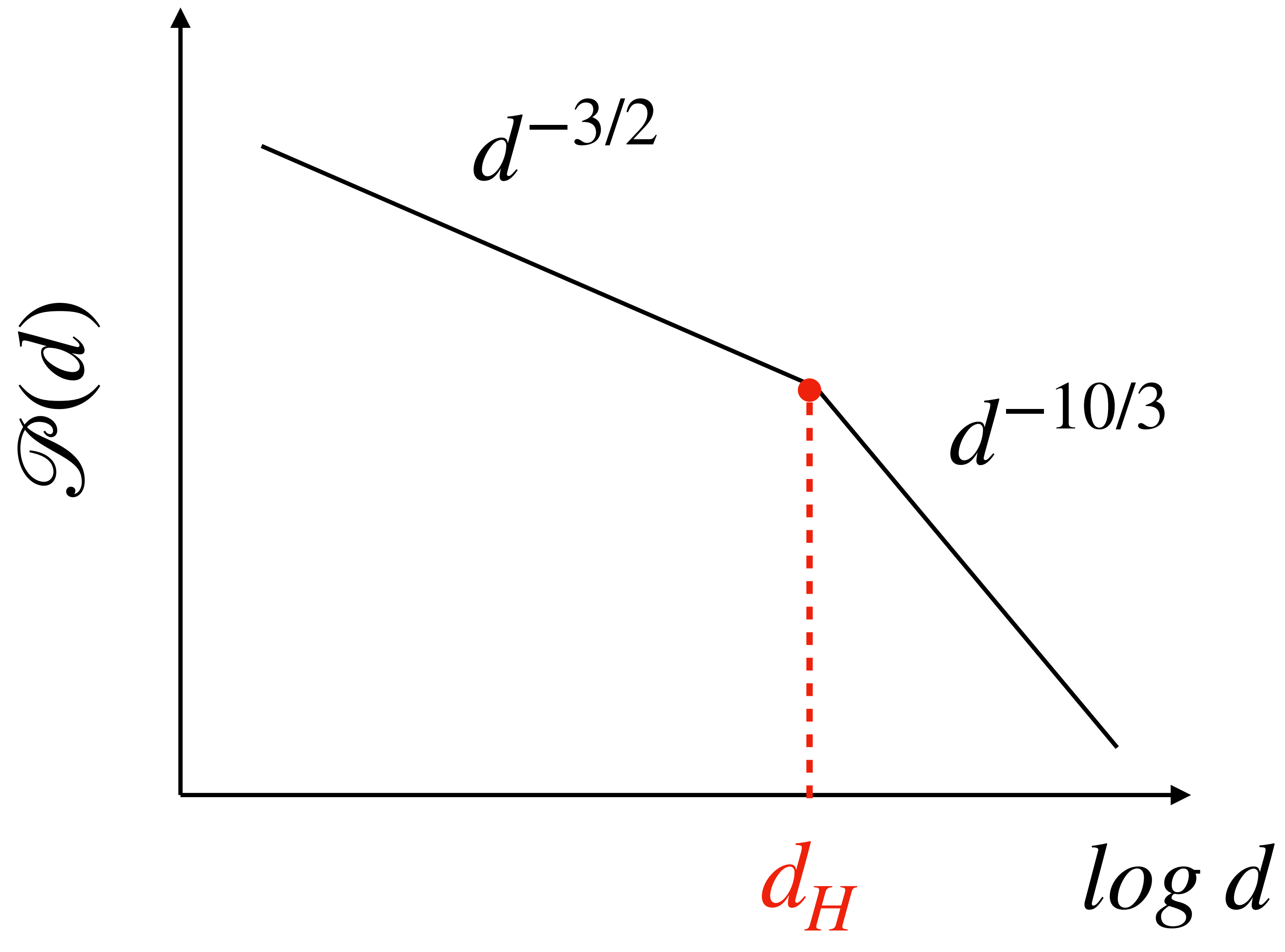
- In general, the presence of the interface generates small-scales agitation, as observed through the energy spectrum
- The presence of the interface creates an alternative path for energy transfer towards small scales
- Surface tension energy transport extends energy exchange at small scales, and it is proportional to the amount of interface present
- The presence of the interface increase small-scale gradients, *i.e.* intermittency on the dissipation and vorticity pdfs
- The presence of the interface is strongly contributing to the decrement of energy transfer through the non-linear term

Droplet dynamics

Crialesi-Esposito, M., Chibbaro, S., & Brandt, L. (2022). **The interaction of droplet dynamics and turbulence cascade**. *Communication Physics 6.1 (2023): 5.*

Crialesi-Esposito, M., Brandt, L., Boffetta G., Chibbaro, S., & Musacchio, S. (2022). **The impact of velocity gradients on droplet formation in multiphase flows**. *In preparation.*

The droplet-size-distribution



[1] Garrett, C., Li, M., & Farmer, D. (2000). The connection between bubble size spectra and energy dissipation rates in the upper ocean. *Journal of physical oceanography*, *30*(9), 2163-2171.

[2] Deane, G. B., & Stokes, M. D. (2002). Scale dependence of bubble creation mechanisms in breaking waves. *Nature*, *418*(6900), 839-844.

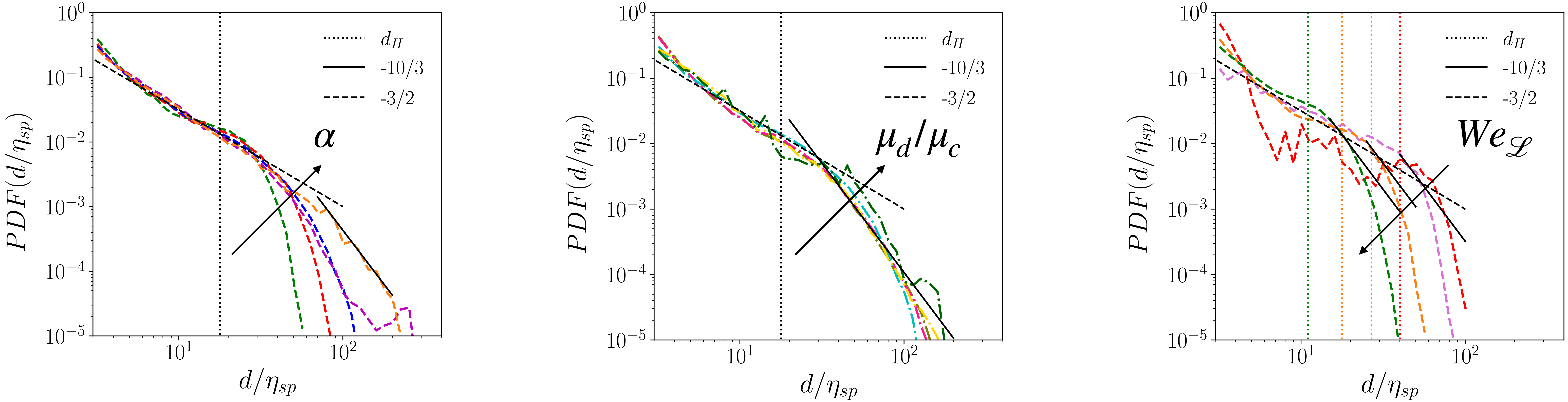
[3] Hinze, J. O. (1955). Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AIChE journal*, *1*(3), 289-295.

- Large droplets, break due to the interaction with turbulence, forming a $d^{-10/3}$ distribution[1]

- As they break, large droplets form satellite droplets, which cannot break further and can only be created by large droplet breakup. These droplets form a $d^{-3/2}$ distribution [2].

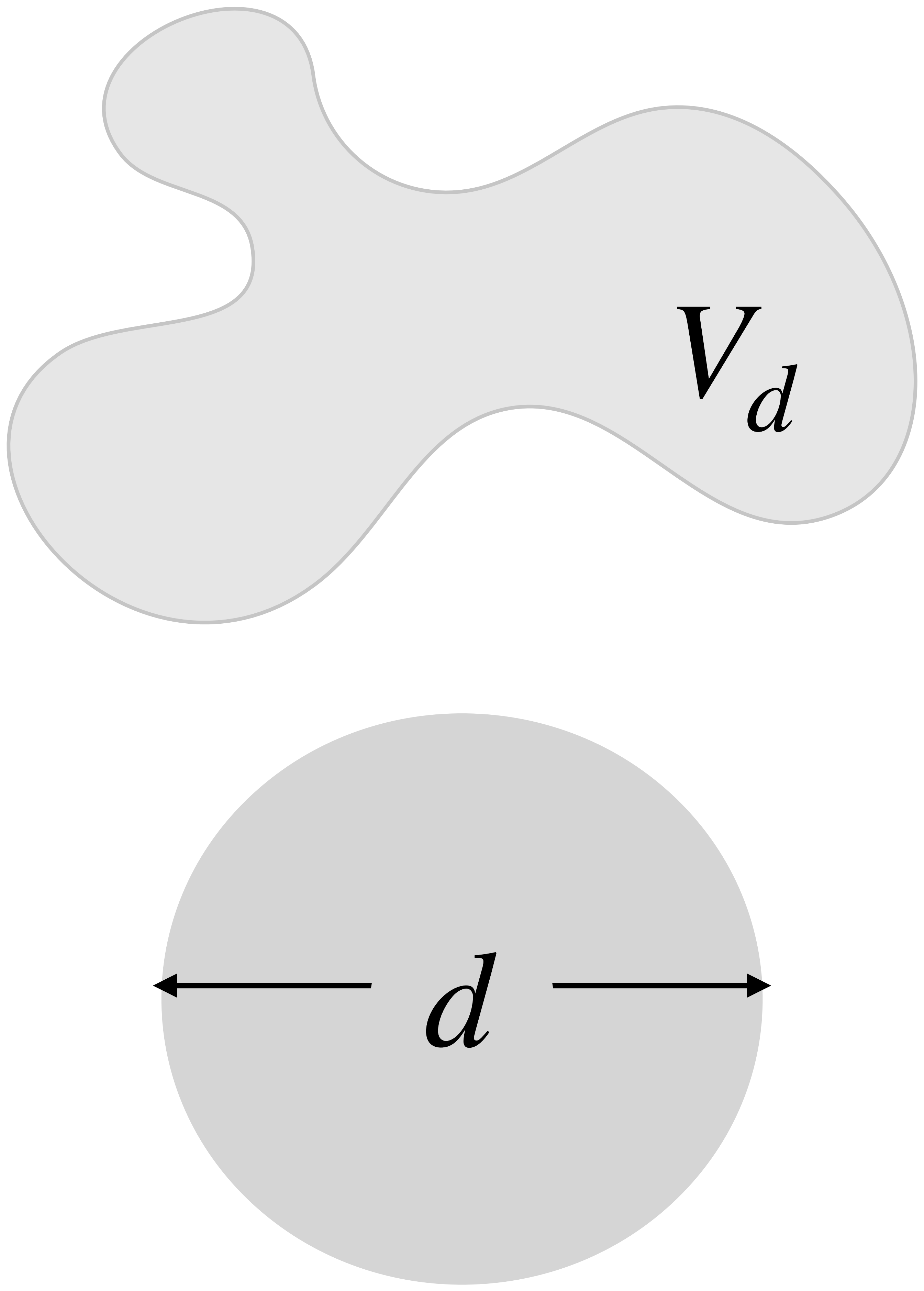
- The size of the largest droplets undergoing breakup is the so-called Hinze-Kolmogorov scale d_H [3]

The droplet-size-distribution



- The −3/2 and −10/3 power-laws are observed in all simulations (with clear exceptions)
- The volume fraction α and $We_{\mathscr{L}}$ have the stronger effect on the distributions
- The canonical Kolmogorov-Hinze scale $d_H = 0.725\varepsilon^{-2/5}(\rho_c/\sigma)^{-3/5}$ seems to poorly predict the crossover between the two power-laws…***can we improve its definition?***

The Kolmogorov-Hinze argument in a nutshell



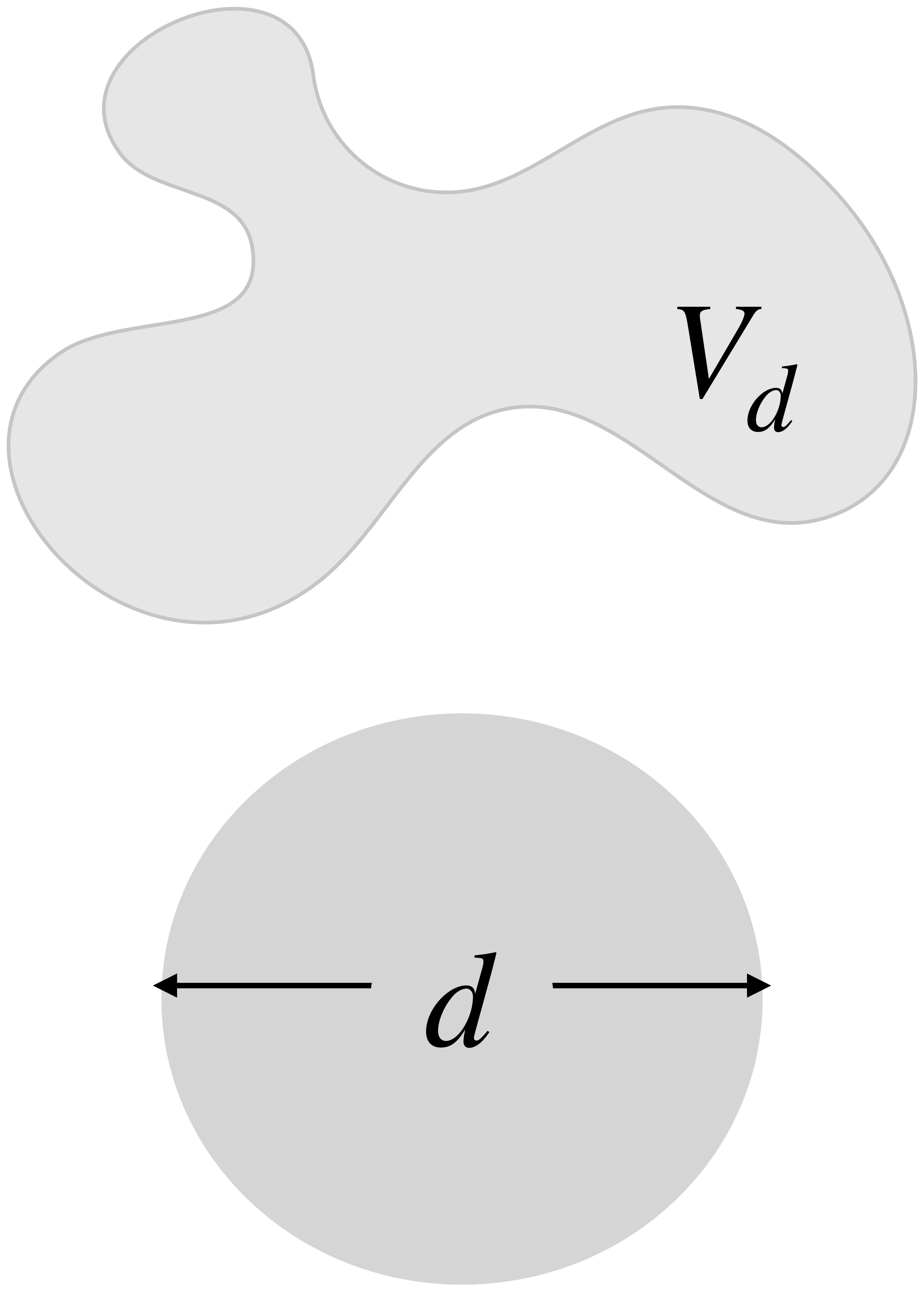
We want to find the size d (*i.e.* characteristic scale) of a droplet undergoing breakup in turbulence

…this translates into a critical Weber number We_c…

$$We_c = \frac{\rho u_d^2 d}{\sigma}$$ and assuming $$u_d \sim u_\ell \sim \langle \varepsilon \rangle^{1/3} d^{1/3}$$

$$d_H = \left(\frac{We_c}{2}\right)^{3/5} \left(\frac{\sigma}{\rho}\right)^{3/5} \langle \varepsilon \rangle^{-2/5}$$

The Kolmogorov-Hinze argument in a nutshell



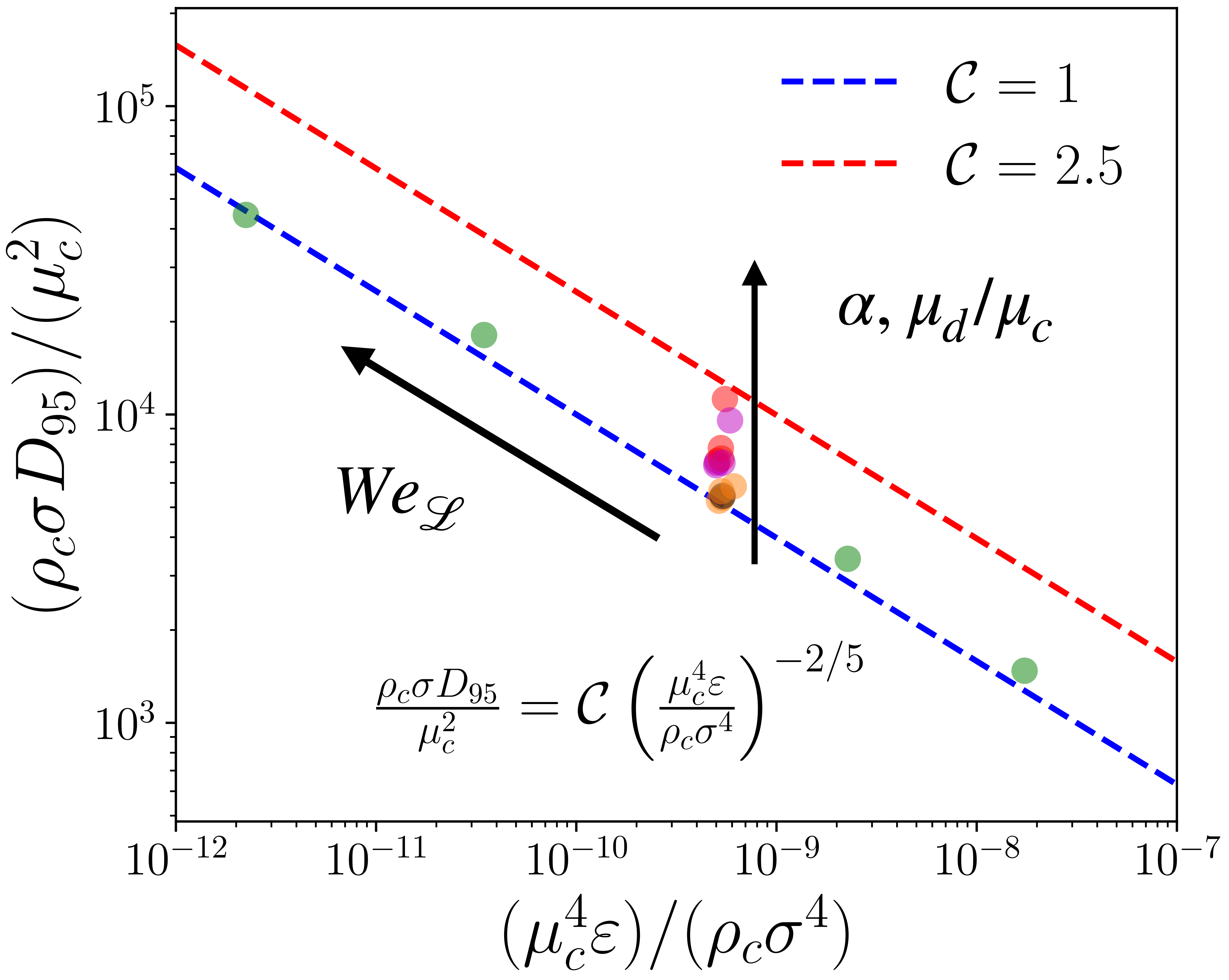
…this approach hides few assumptions…

- $d \sim \ell$ is in the inertial range, where $u_d \sim \varepsilon^{1/3} d^{1/3}$ holds, as $\Pi \approx \varepsilon$
- ε is a system invariant and it is not strongly modulated by the presence of the interface
- We_c is a *non-universal* constant

$$d_H = \left(\frac{We_c}{2}\right)^{3/5} \left(\frac{\sigma}{\rho}\right)^{3/5} \langle \varepsilon \rangle^{-2/5}$$

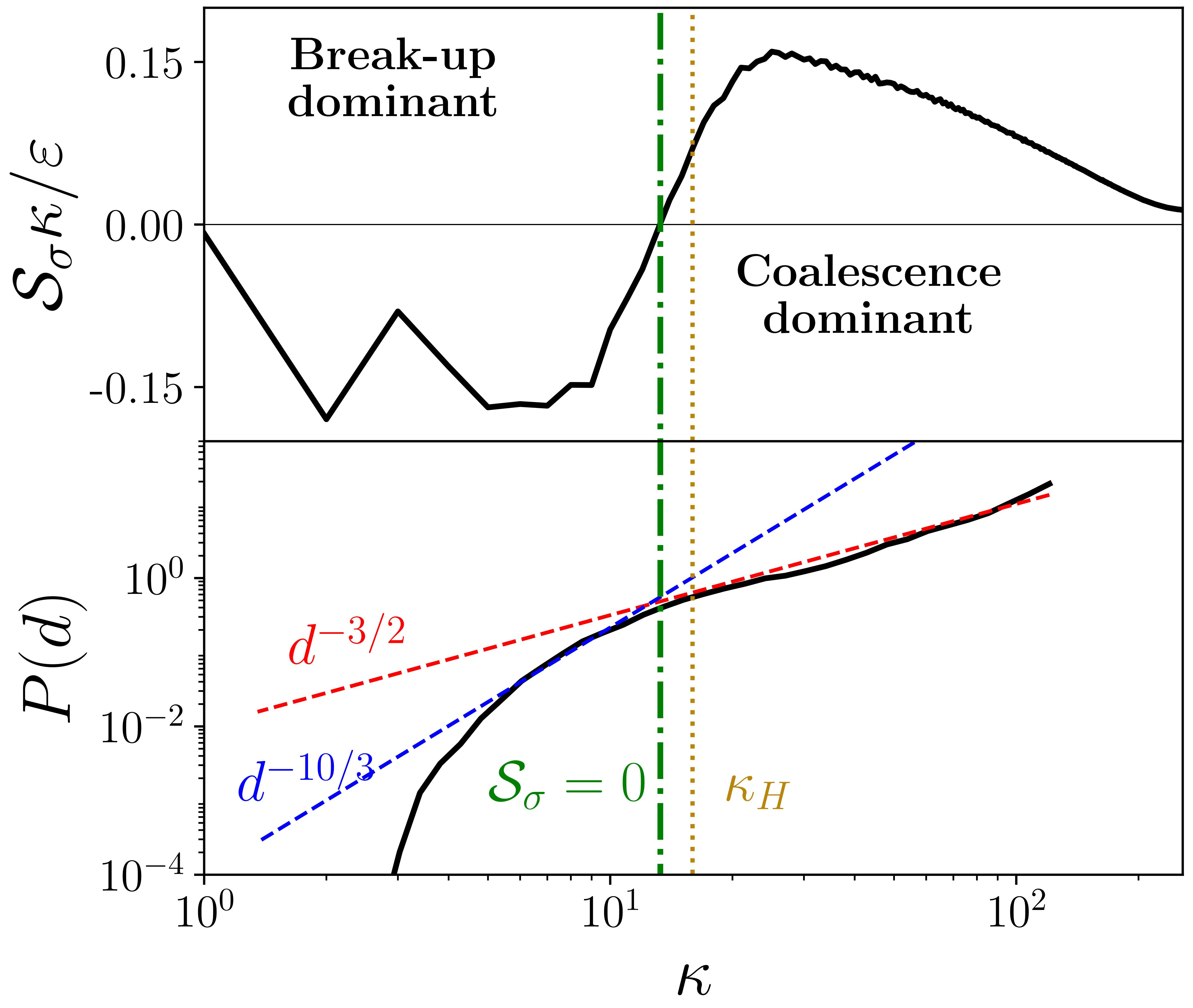
The criticality of the critical Weber number We_c

$$C = \left(\frac{We_c}{2}\right)^{3/5}$$



- While varying $We_{\mathscr{L}}$, a single value of We_c can be used. (Note that $\alpha = 0.03$)
- While varying α and μ_d/μ_c the coefficient C changes significantly
- Larger variations are also observed in literature

The Kolmogorov-Hinze as the crossover scale

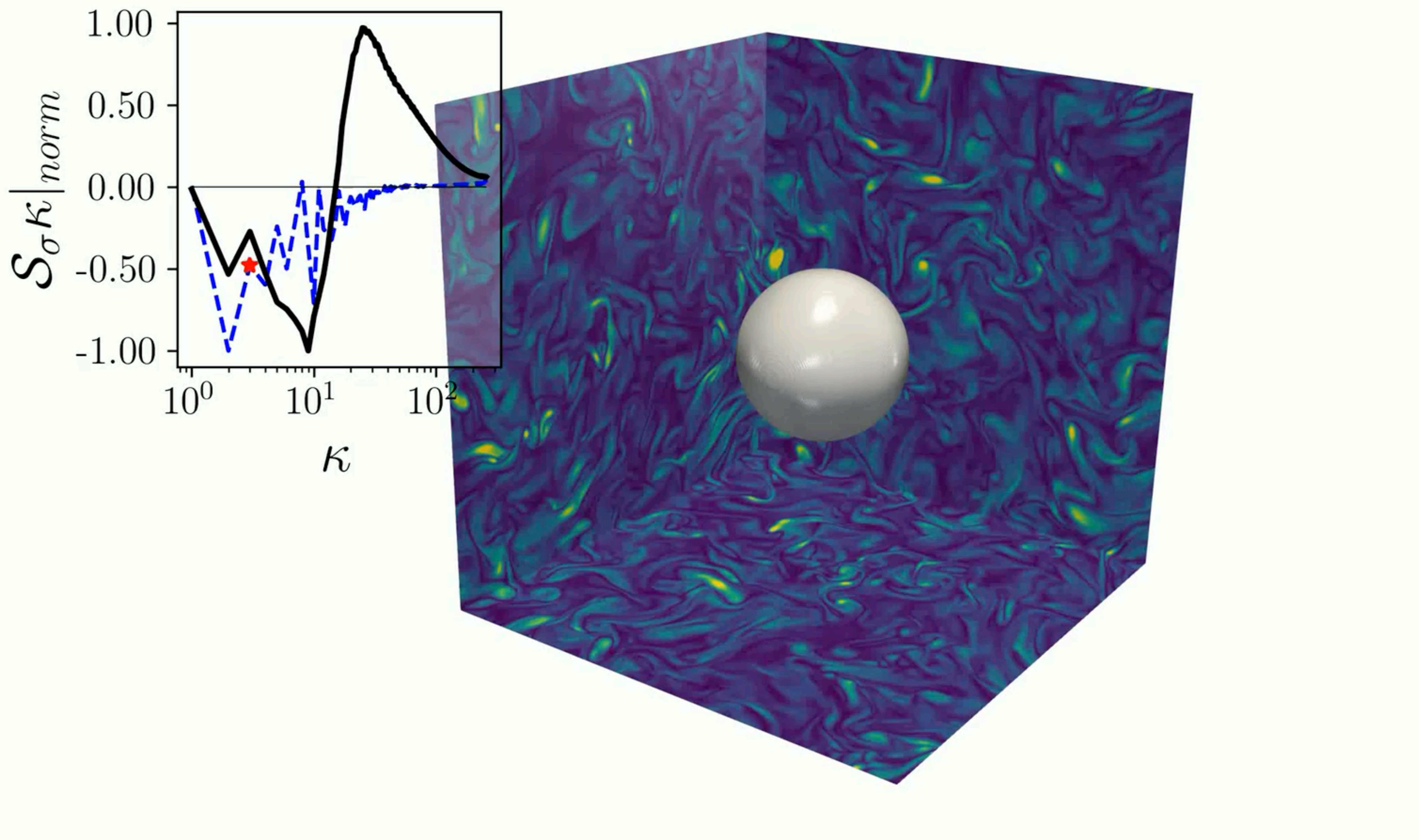


- The surface tension energy transfer term $\mathcal{S}_\sigma$ absorbs energy at large scale and reinject energy at small scales.
- The scale at which the net energy transferred through the interface is zero coincide with the crossover scales for the power-laws in the DSD
- The crossover also marks two regions, each dominated either by breakup or coalescence

Does it hold also in a deterministic sense?

Single droplet breakup dynamic

https://static-content.springer.com/esm/art%3A10.1038%2Fs42005-022-01122-8/MediaObjects/42005_2022_1122_MOESM3_ESM.m4v



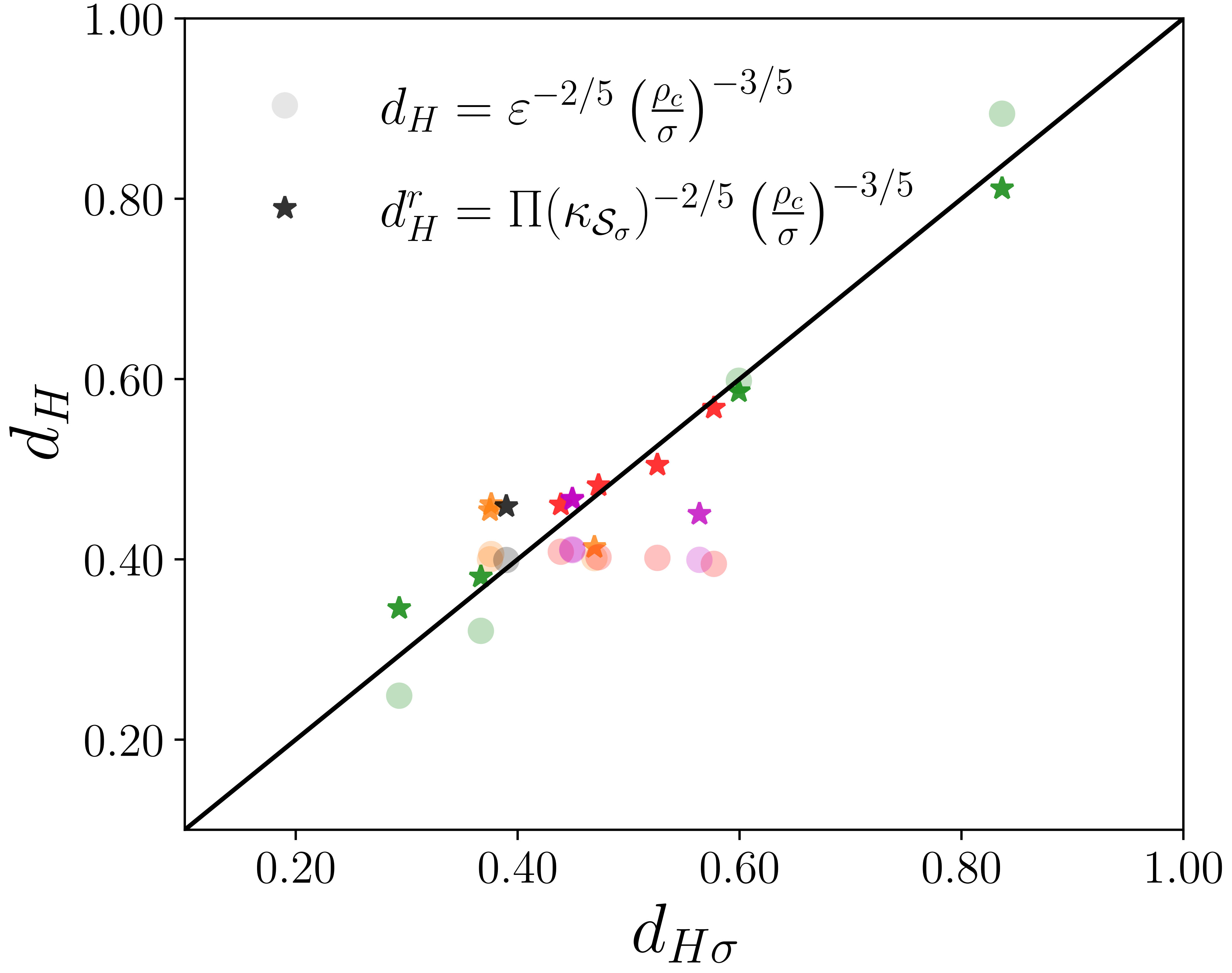
What have we learned so far?

- The presence of the interface alters significantly the energy transport mechanism, hence $\Pi < \varepsilon$ in the inertial range

- The scale at which the net energy transferred through the interface is zero coincide with the crossover scales for the power-laws in the DSD. Here, the energy transferred through the interface is maximum

- So far, everything hints that the breakup process is strongly scale-local in a statistical sense, and that ε is not a good estimator for the energy flux

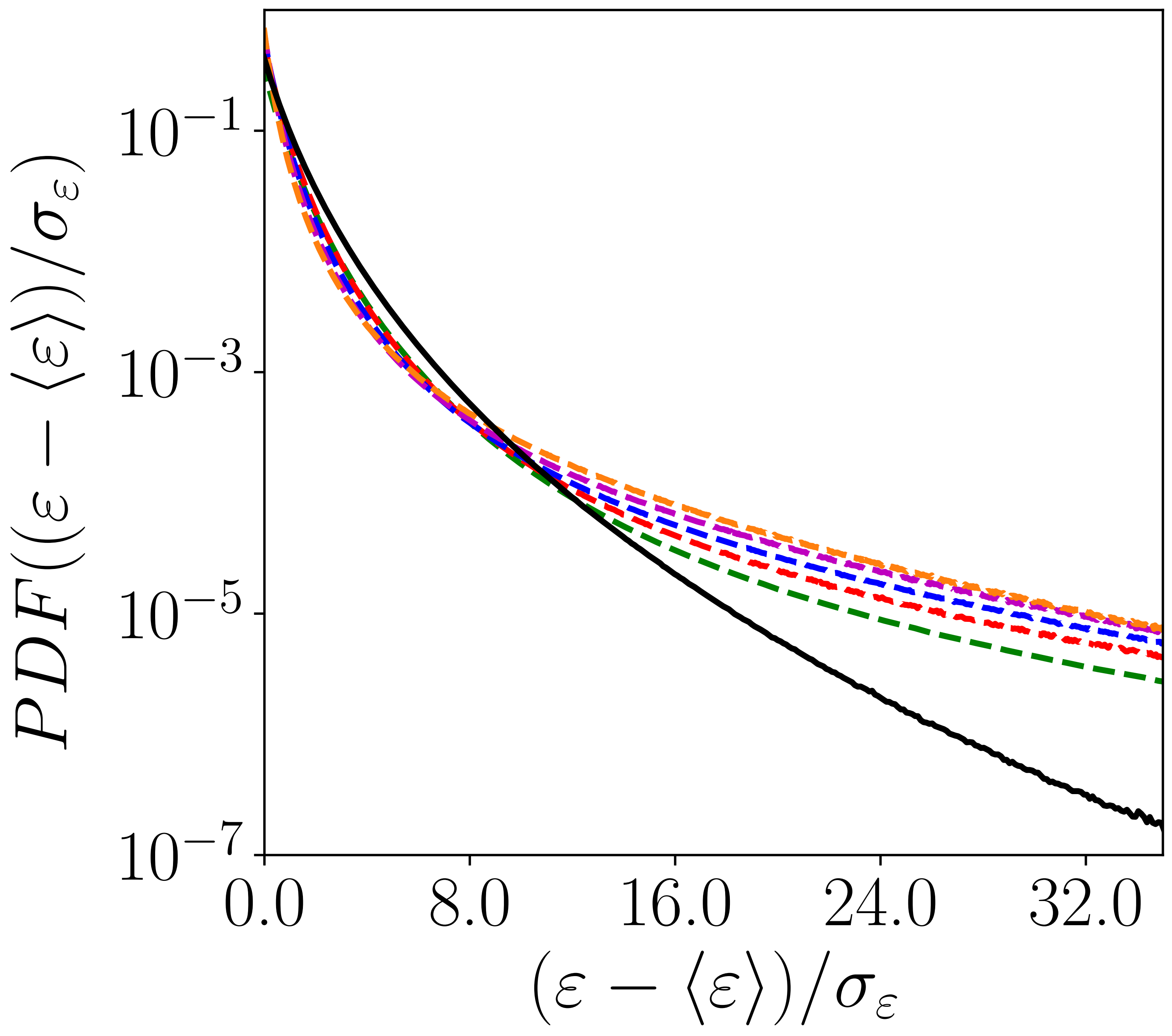
Revisiting the Kolmogorov-Hinze theory



d_{H_σ} is the scale where $S_\sigma = 0$

- If the correct KH scale is at the crossover, at κ_{S_σ} where $S_\sigma = 0$, then the energy flux should be evaluated at such scale
- By using $\Pi(\kappa_{S_\sigma})$, the error decrease significantly, especially at different α (red symbols)
- A 0.8 pre-factor is applied, which is likely due to finite-Re effects (at $Re_\lambda < 200$ a fully developed inertial range is not visible)

The role of energy dissipation



$$d_H = \left(\frac{We_c}{2}\right)^{3/5} \left(\frac{\sigma}{\rho}\right)^{3/5} \langle\varepsilon\rangle^{-2/5}$$

↓

$$d = a \sim \varepsilon^{-2/5}$$

and

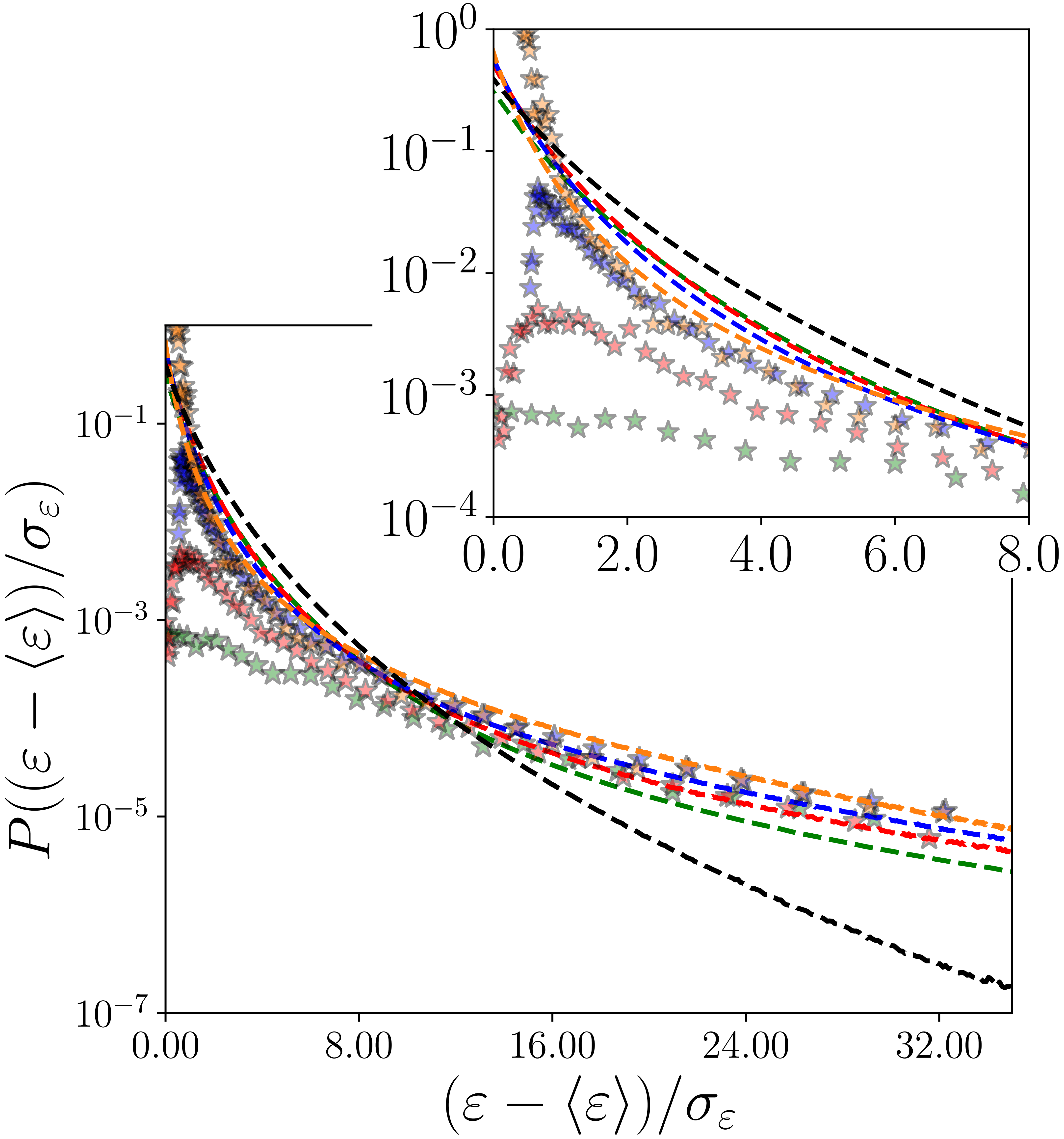
$$P(a)da = P(\varepsilon)d\varepsilon$$

↓

$$P(\varepsilon) \sim a^{13/2} N(a)$$

- In general, we can assume that $d_h(\langle\varepsilon\rangle)$
- We previously observed an increment in the dissipation intermittency, i.*e.* regions of high dissipation.
- Will d and ε still correlate in those regions in a statistical sense?

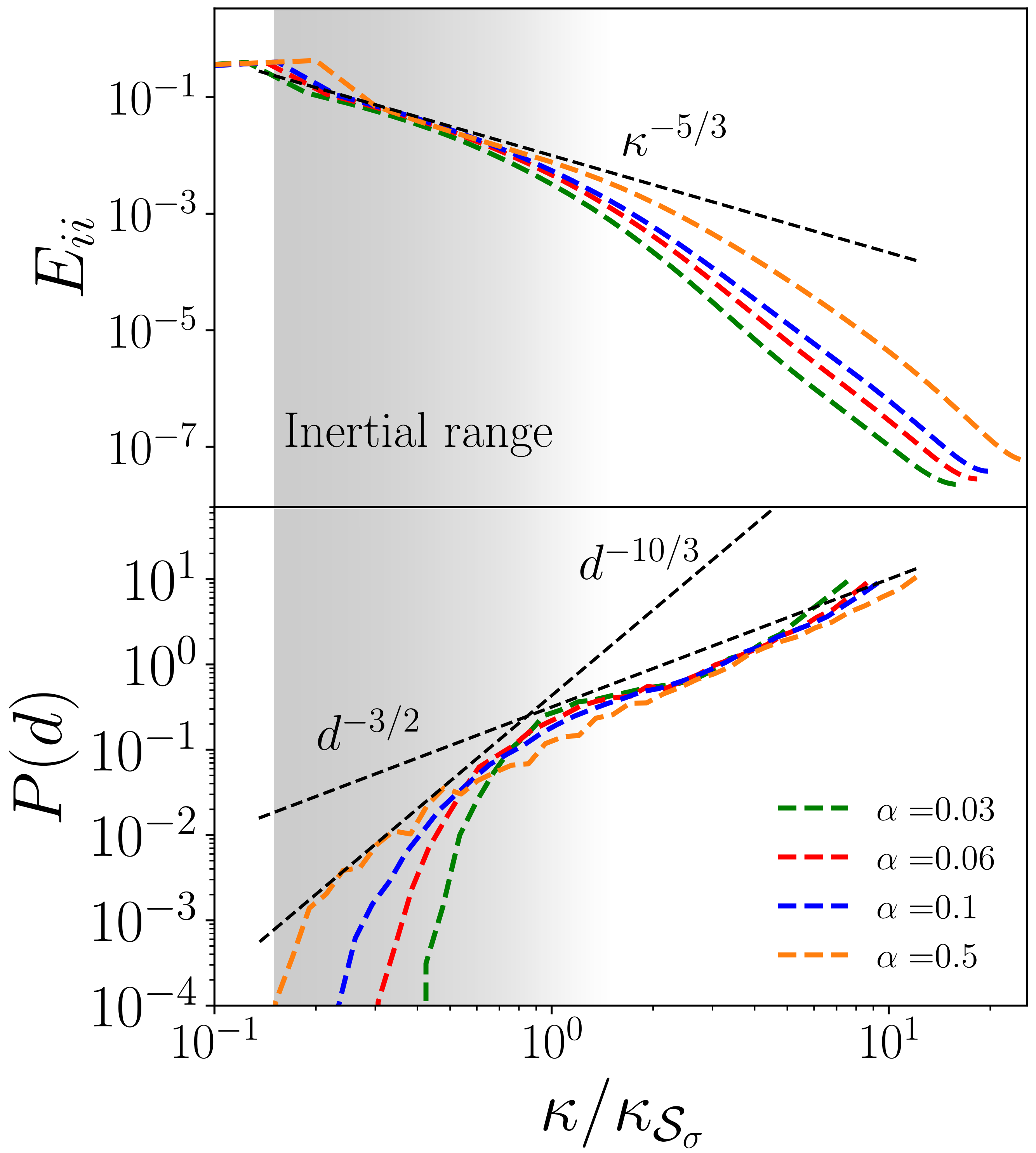
The role of energy dissipation



- Converting the DSD to $P(\varepsilon)$, the data collapse on the original pdf
- At large ε, small d, the distribution match
- At small ε, large d, better agreement is obtained at large volume fractions

$$P(\varepsilon) \sim d^{13/2} N(d)$$

$$d \sim \varepsilon^{-2/5}$$



Does intermittency relates to the production of small droplets?

$$P(\varepsilon) \sim d^{13/2} N(d)$$

$$d \sim \varepsilon^{-2/5}$$

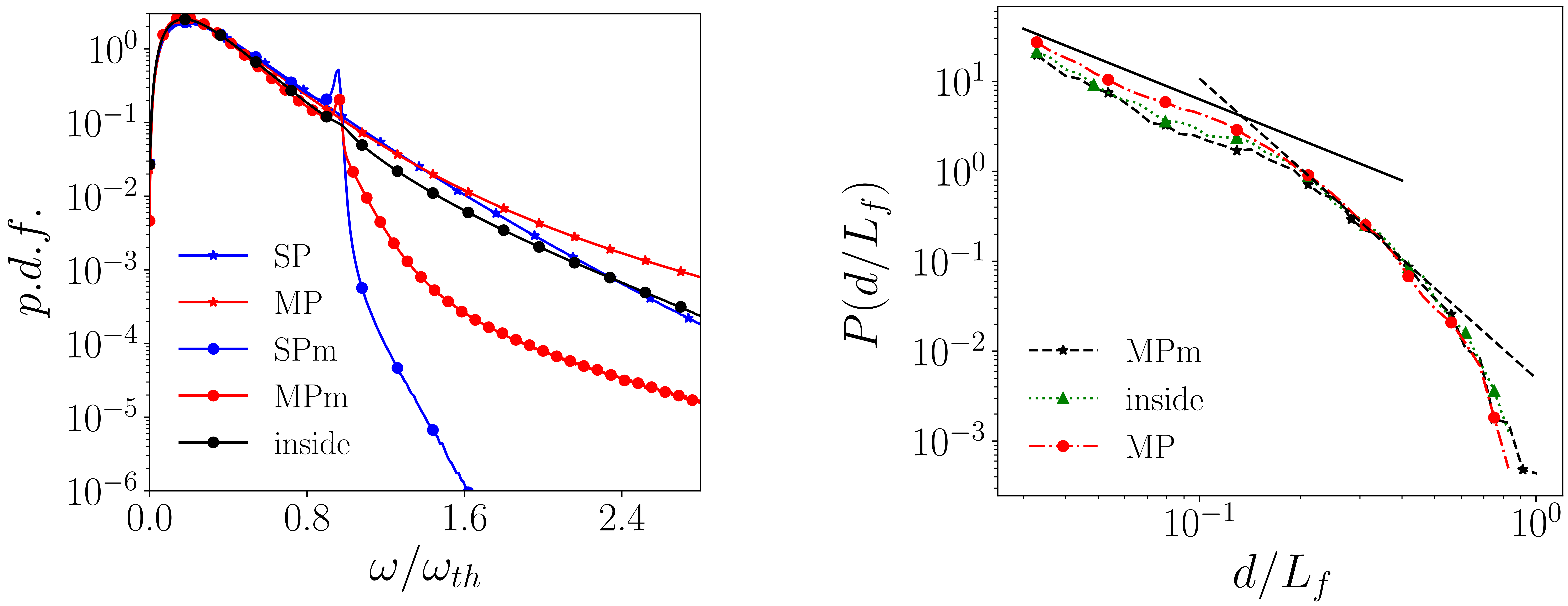
Buzzicotti Michele, **Luca Biferale**, and Federico Toschi. "*Statistical properties of turbulence in the presence of a smart small-scale control*." *Physical Review Letters* 124.8 (2020): 084504.

$$\frac{d\mathbf{u}}{dt} = -\nabla P + \nabla \cdot \left[\nu([\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})\right] + \sigma \xi \delta_s \mathbf{n} + \mathbf{f} + \mathbf{f}^{\mathbf{C}}$$

$$\mathbf{f}^{\mathbf{C}} = -\mathbf{u}C = \beta \left(\frac{tanh(\omega - \omega_{th}) + 1}{2} \right)$$

$$\omega_{th} \approx 5\sigma_\omega$$

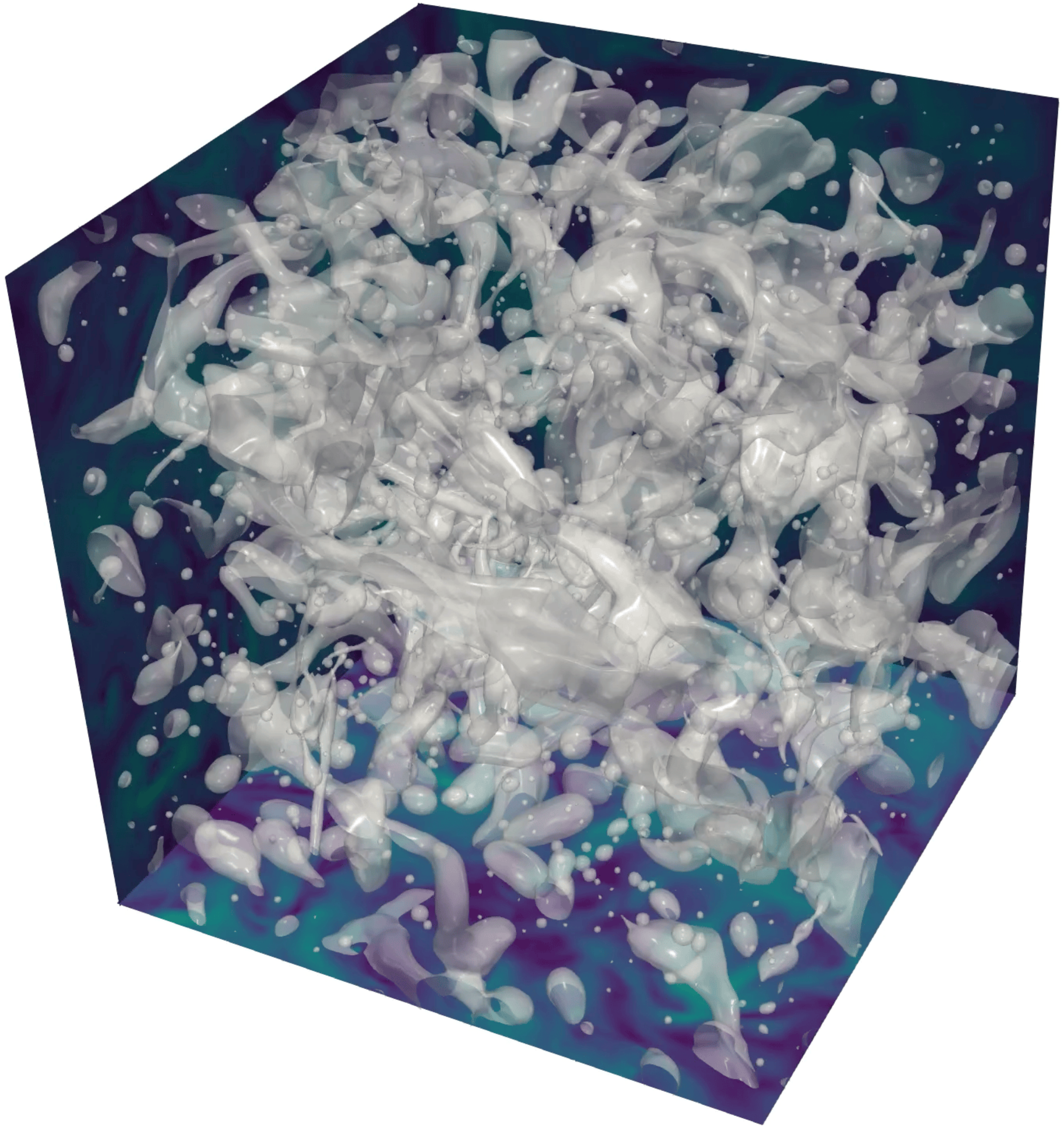
Does intermittency relates to the production of small droplets?



Conclusions

- We assess that there is a tight correlation between the droplet formation and the modulation of energy transport across scales
- The Kolmogorov-Hinze scale can be found at the scale where surface tension net energy transfer is null.
- The formation of small droplets tightly correlates to regions of high dissipation, hence is tightly coupled to intermittency
- The Kolmogorov-Hinze theory could be revised by using the value of the energy fluxes at the scale where surface tension energy transfer is zero

Thanks for your attention!



And to all my collaborators:

Luca Brandt, Sergio Chibbaro, Marco Rosti, Stefano Musacchio, Guido Boffetta

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